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Plane Symmetric String Cosmological Model in Modified Theory of General Relativity

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Abstract It is shown that homogeneous plane symmetric string cosmological model for Takabayasi string i.e. $\rho = (1+\omega)\lambda$ does not exist in Barber's second self creation theory. Further it is found that the string cosmological model in this theory exist only when $\omega = 0$. Therefore model for $\rho = \lambda$ (geometric string) is constructed. Some physical and geometrical properties of the model are discussed.

Keywords Plane symmetry · Cosmic string · Self-creation theory

1 Introduction

In recent years researchers have lot of interest in the cosmological consequences of line like topological defects called cosmic strings, which may be produced during phase transition in the early universe [13, 15, 40, 41, 43]. Such strings would produce density fluctuations on very large scales and may be responsible for the formation of galaxies [14]. Zeldovich [47] and Vilenkin [42] studied the evolution of the network of strings formed in the early universe and pointed out that the gravitational effects of cosmic strings are responsible for creation of galaxies and clusters. Letelier [19] and Stachel [34] studied the gravitational effect of cosmic strings in general theory of relativity. Letelier [20] constructed string cosmological models in Bianchi type I and Kantowski–Sachs spacetimes. Banerjee et al. [3] constructed Bianchi type I string cosmological models in the presence and absence of a source free magnetic field. Krori et al. [17] obtained exact solutions of the Einstein's field equations for Bianchi type II, VI_0, VII and IX spacetimes in the presence of cosmic strings. Chakraborty [10, 11], Chakraborty and Chakraborty [12], Nevin [24], Tikekar and Patel [36], Ram and Singh [27], Bhattacharya and Karade [6], Tikkekar et al. [37], Kilinc and Yavuz [16], Yavuz and Tarhan [46], Baysal et al. [5], Bali and Dave [1], Singh [31], Bali and Upadhaya [2] and Wang

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[44, 45] are some of the authors who have investigated various aspects of cosmic strings in general theory of relativity. Moreover Krori et al. [18] pointed out that cosmic strings do not occur in Bianchi type-V space-time.

Einstein's General theory of relativity has many controversies. Therefore many authors proposed various alternative theories by modifying the general theory of relativity. Barber [4] proposed two theories called self creation theories. His first theory is a modification of Brans and Dicke [8] theory and second theory is an adaptation of general relativity to a variable G-theory. His first theory is inconsistent as it violates equivalence principle [7]. But in view of the consistency of his second theory, many authors [9, 21–23, 25, 26, 28–30, 32, 33, 39] studied various aspects of this theory in presence of different gravitating fields. Recently Venkateswarlu et al. [38] constructed Bianchi-I, II, VIII and IX string cosmological models in Barber's second self creation theory.

In this paper we have taken an attempt to construct plane symmetric string cosmological models in Barber's second self creation theory. We found that Takabayasi strings are not compatible in Barber's second self creation theory for this space time. Therefore the model for $\rho = \lambda$ is constructed.

2 Field Equations

In this section we consider the plane symmetric metric of the form

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2, \tag{1}$$

where A and B are functions of cosmic time t only.

The field equations in Barber's second self creation theory are

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R = \frac{-8\pi T_{ij}}{\phi} \tag{2}$$

and

$$\Box\phi = \frac{8}{3}\pi\eta T, \tag{3}$$

where η is coupling constant to be evaluated from experiment and ϕ is the Barber's scalar. Here ϕ is a function of t. In the limit $\eta \to 0$ the theory approaches Einstein's theory in every respect.

The energy momentum tensor for a cloud of massive strings given by Letelier [19, 20] and Stachel [34] is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j. \tag{4}$$

Here ρ is the rest energy density for a cloud of strings with particles attached to them, λ is the string tension density, u^i is the four velocity for the cloud of particles, x^i is the four vector which represents the strings direction which is the direction of anisotropy and

$$\rho = \rho_p + \lambda, \tag{5}$$

where ρ_p denotes particle energy density.

Moreover the direction of strings satisfy the standard relations

$$u^i u_i = -x^i x_i = 1, u^i x_i = 0. \tag{6}$$

By using the co-moving co-ordinate system and (4) and (6) the field equations (2) and (3) for the metric (1) yield

$$\left(\frac{A'}{A}\right)^2+\frac{2A'B'}{AB}=8\pi\phi^{-1}\rho, \tag{7}$$

$$\frac{A''}{A}+\frac{A'B'}{AB}+\frac{B''}{B}=0, \tag{8}$$

$$2\frac{A''}{A}+\left(\frac{A'}{A}\right)^2=8\pi\phi^{-1}\lambda \tag{9}$$

and

$$\phi''+\left(\frac{2A'}{A}+\frac{B'}{B}\right)\phi'=\frac{8}{3}\pi\eta\left(\rho+\lambda\right). \tag{10}$$

Here afterwards the dash over the field variable represents ordinary differentiation with respect to time.

3 Solutions and Models

The system of field equations (7)–(10) is an under determined system and to make the system consistent we consider

$$\rho=(1+\omega)\lambda,\quad \omega>0\quad \text{(Takabayasi string [35])}. \tag{11}$$

From (7), (9) and (11) we obtain

$$\frac{A'}{A}\left(\frac{2A''}{A'}+\frac{\omega}{1+\omega}\frac{A'}{A}-\frac{2}{1+\omega}\frac{B'}{B}\right)=0, \tag{12}$$

which yields following cases

$$\text{Case I:}\quad A'=0,$$

$$\text{Case II:}\quad \frac{2A''}{A'}+\frac{\omega}{1+\omega}\frac{A'}{A}-\frac{2}{1+\omega}\frac{B'}{B}=0.$$

Case I leads to vacuum model obtained by Mohanty et al. [21]. For Case II we find

$$A=at+a_1, \tag{13}$$

$$B=a_2\left(at+a_1\right)^{\frac{\omega}{2}}, \tag{14}$$

$$\frac{8\pi}{\phi}\rho=\frac{a^2\left(1+\omega\right)}{\left(at+a_1\right)^2} \tag{15}$$

and

$$\frac{8\pi}{\phi}\lambda=\frac{a^2}{\left(at+a_1\right)^2}, \tag{16}$$

where $a(\neq 0)$, a_1 and $a_2(\neq 0)$ are constants of integration.

Substituting (13) and (14) in (8) we find

$$\omega = 0. \tag{17}$$

Thus the Takabayasi's equation of state is not compatible and we get

$$\rho = \lambda. \tag{18}$$

The set of solution for geometric strings or Nambu strings [19] is obtained as

$$A = at + a_1, \tag{19}$$

$$B = a_2, \tag{20}$$

$$\frac{8\pi}{\phi}\rho = \frac{8\pi}{\phi}\lambda = \frac{a^2}{(at + a_1)^2}. \tag{21}$$

In this case (10) reduces to

$$(at + a_1)^2\phi'' + 2a\,(at + a_1)\,\phi' - \frac{\eta a^2}{3}\phi = 0, \tag{22}$$

which yields the solution

$$\phi = c_1(a_1 t + a_2)^{m_1} + c_2(a_1 t + a_2)^{m_2}, \tag{23}$$

where c_1 and c_2 are constants of integration and

$$m_1 = \frac{-1 + \sqrt{1 + \frac{4}{3}\eta}}{2}, \qquad m_2 = \frac{-1 - \sqrt{1 + \frac{4}{3}\eta}}{2}.$$

Thus the geometry of the universe described by the line element with suitable transformation is

$$ds^2 = dT^2 - T^2(dx^2 + dy^2) - dz^2. \tag{24}$$

The metric (24) represents string cosmological model for geometric strings (Nambu strings) in Barber's second self creation theory.

4 Some Physical and Geometrical Properties of the Model

The model (24) represents geometric string cosmological model in Barber's second self creation theory. This model possesses singularities at initial epoch and infinite future. The behaviors of the physical and kinematical variables involved in this universe are given as follows:

(a) The Barber scalar ϕ is obtained as

$$\phi = T^{m_1} + T^{m_2},$$

where $m_1 = \frac{1}{2}(-1 + \sqrt{1 + \frac{4}{3}\eta})$, $m_2 = -\frac{1}{2}(1 + \sqrt{1 + \frac{4}{3}\eta})$. Here $\phi \to 0$ as $T \to 0$ or $T \to \infty$. Hence at initial epoch and at infinite future the theory leads to general theory of relativity. Further when the coupling constant $\eta \to 0$ one mode of ϕ i.e. $T^{m_1} \to 0$ whereas the other one T^{m_2} does not approaches zero. Therefore the later mode of ϕ is not acceptable.

(b) The energy density (tension density)

$$8\pi\rho(=8\pi\lambda) = \frac{T^{m_1} + T^{m_2}}{T^2}$$

satisfy the energy condition $\rho \geq 0$.

(c) The spatial volume of the model is obtained as

$$V = T^2.$$

Hence at the initial epoch volume of the universe is zero. As time increases the volume increases and $V \to \infty$ as $T \to \infty$.

(d) The scalar expansion θ is calculated as

$$\theta = \frac{6}{T}$$

from which it is evident that the rate of expansion of the universe becomes slow as time increases.

(e) The shear scalar for the model is given by

$$\sigma^2 = \frac{1}{6}T^2.$$

Since $\sigma^2 \to \infty$ as $T \to 0$ and $\sigma^2 \to 0$ as $T \to \infty$ the shape of the universe changes uniformly in x and y directions only. The rate of change decreases with increase of time.

(f) It is observed that $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, which indicates that the universe is anisotropic for large T.

(g) The deceleration parameter q is obtained as $q = -\frac{1}{2}$. The negative value of q indicates that the model is inflationary.

5 Conclusion

In this paper it is shown that the cosmological model for Takabayasi strings does not exist in Barber's second self creation theory in plane symmetric space-time, because the equation of state for Takabayasi string viz. $\rho = (1+\omega)\lambda$ admits model only when $\omega = 0$. Therefore we constructed a model for geometric strings i.e. when $\omega = 0$. The model (24) is inflationary. If we take $A = B$ in the metric presented by Venkateswarlu et al. [38] the present model can be obtained but it is not studied in their paper. The solutions obtained in this paper are more general than those obtained by Venkateswarlu et al. [38].

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